LARGE ANGLE MAGNETIC SUSPENSION

TEST FIXTURE

Grant NAG-1-1056, ODU 194475

Principal Investigator - Dr. Colin P. Britcher, Department of Aerospace Engineering

Co-Investigator - Dr. Jen-Kuang Huang, Department of Mechanical Engineering

For attention of Nelson J. Groom
Guidance and Control Branch
Flight Dynamics and Control Division
MS 161
NASA Langley Research Center

Final Report for the Period 11-1-95 thru 10-31-96

SUMMARY

Progress was made this year in several major areas. These include eddy current computations, modelling and analysis, design optimization methods, wind tunnel Magnetic Suspension and Balance Systems (MSBS), payload pointing and vibration isolation systems, and system identification. In addition, another successful International Symposium was completed, with the Proceedings edited and published. A 4th Symposium has been planned and preparations are well in hand.

These activities continued and concluded several years of work under this Grant and extended previous work on magnetic suspension systems and devices in the Guidance and Control Branch. Research accomplishments facilitated the demonstration of several new developments in the field of magnetic suspension technology.

This report concentrates on the period 11/1/95 thru 10/31/96, previous periods having already been the subject of progress reports. A summary of all Grant activity is provided at the conclusion of this report.

REVIEW OF WORK DURING THE SUBJECT PERIOD

(i) Eddy current modelling. The ELEKTRA computer code has been used to calculate forces, stored energy, field magnitude and phase, and power losses for magnetic suspension (LAMSTF-like) and magnetic bearing (ASPS-like) configurations. Various problems have been encountered and overcome, such that an analysis capability suitable for application to the LGMSS project has steadily emerged. The current status of this work is reflected in some of the publications listed later in this report.

(ii) Design optimization. An effort to apply state-of-the-art optimization methods and computer codes to the magnetic suspension problem has begun. Initial analysis has concentrated on small-gap, axisymmetric bearings. It has been shown that optimum designs based on maximum force, minimum power, etc., are identifiable and are distinctly different from each other. This effort will result in a publication at some point in the future. Analysis will then proceed to the large-gap problem, which is more challeging, although some work has already been accomplished (by David Cox, LaRC). This work will definately extend beyond the conclusion of this Grant.

(iii) Wind Tunnel Magnetic Suspension and Balance Systems. There appears to be continuing interest in this application, both in general, and for a specific test objective, namely ultra-high Reynold's number testing. Work on recommissionig the ex-MIT, ex-NASA, 6-inch MSBS at ODU continues, but at a very slow pace due to lack of funding and documentation. Alternate funding sources for future work are being explored. Two recent presentations concerning the wind tunnel application have been made and copies are attached as Appendices to this report.

(iv) Annular Suspension and Pointing System. Work continues at a low level, following successful levitation in five degrees-of-freedom. The control software has been "cleaned up" and some electrical upgrades made to reduce noise. A second joint Proposal for future work with LaRC and Boeing has been submitted and would result, if successful, in a dramatic increase in effort in this area.

(v) Symposium support. Support was provided for the organization and execution of the 3rd International Symposium on Magnetic Suspension Technology in Tallahassee, Florida. This meeting was a great success with no significant problems encountered. A 4th Symposium has been planned for late 1997, in Gifu City, Japan. The P.I. will continue to provide support for this meeting beyond the end of the current Grant.

(vi) System Identification. This Section is submitted by the Co-Investigator.

FINAL REPORT

IDENTIFICATION AND CONTROL OF MAGNETIC SUSPENSION SYSTEMS

SUMMARY

For identifying a dynamic system, operating under a stochastic environment, projection filters, which were originally derived for deterministic systems, are developed by using optimal estimation theory. This newly developed system identification algorithm is successfully implemented at NASA Langley Research Center for identification of unstable large-gap magnetic suspension systems. The results show that it can be applied for dynamic systems under closed-loop operation with known or unknown feedback dynamics. The test data processed can be either in time domain or frequency domain. It is also very effective to be used for controller design for nonlinear unstable systems and for direct Kalman filter gain estimation without knowing noise covariances.

This report summarizes the indirect closed-loop time-domain system identification algorithm and an iterative LQG controller redesign cycles developed for magnetic suspension systems. In each cycle, the closed-loop identification method is used to identify an open-loop model and a steady-state Kalman filter gain from closed-loop input/output test data obtained by using a feedback LQG controller designed from the previous cycle. Then the identified open-loop model is used to redesign the state feedback. The state feedback and the identified Kalman filter gain are used to form an updated LQG controller for the next cycle. This iterative process continues until the updated controller converges. The proposed indirect closed-loop system identification and controller design is demonstrated by numerical simulations and experimental results.

1 Introduction

Classical Linear Quadratic Gaussian (LQG) controllers are designed by solving two separate, but dual problems: the Linear Quadratic Regulator (LQR) design and Kalman filter (i.e., optimal state estimator) design. The performance of the controllers relies on an accurate open-loop model for the LQR and an accurate estimate of the measurement and process noise statistics for the Kalman filter. It is difficult to obtain an accurate model through analysis for some systems, and an accurate estimate of the noise statistics through testing for most systems. Furthermore, the noise statistics may be related to the controller if part of the measurement and process noise are generated by the sensor and actuator amplifiers, respectively. To overcome these problems, we present an iterative LQG controller design approach for a linear stochastic system with an uncertain open-loop model and unknown noise statistics. This approach consists of closed-loop identification and controller redesign cycles. The closed-loop identification method can simultaneously identify the open-loop model and the Kalman filter gain under the closed-loop operation with a known dynamic controller. Then the identified open-loop model is used for the LQR design. The LQR and the identified Kalman filter gain are used to form the updated LQG controller for the next closed-loop identification. The process continues until the updated LQG controller converges.

For system identification, several methods (Chen et al., 1992a, 1992b, 1993; Phan et al., 1991; Juang et al., 1993) have been introduced recently to identify the state-space model of a linear system and the Kalman filter. Typically the system is under open-loop excitation with an uncorrelated white noise input. For an unstable system, the input/output data are not available while it is under an open-loop operation. To directly use these methods, we have to design a controller and an input signal for the closed-loop system so that the input signal to the open-loop system is almost white. Unfortunately, this is very difficult. On the other hand, some identification methods (Phan et al., 1992; Liu and Skelton, 1990) have been proposed recently for identifying a system under closed-loop operation. However, they have several shortcomings. First, the Kalman filter gain can not be simultaneously identified because they are applied only for *deterministic* systems. In Phan et al. (1992), no recursive form was derived for computing the open-loop system Markov parameters, and in Liu and Skelton (1990), the approach is based on system *pulse* response. In this report, a *recursive* form for computing the open-loop system and Kalman filter Markov parameters is derived for *stochastic* systems with *random* excitation.

For a system under closed-loop operation, a novel approach for identifying the open-loop model and Kalman filter gain is presented. First, we derive the relation between closed-loop state-space and AutoRegressive with eXogeneous (ARX) models for stochastic systems. From the derivation, it can be seen that a state-space model can be represented by an ARX model if the order of the ARX model is chosen large enough. Since the relation between the input/output data and the system parameters of an ARX model is linear, a linear programming approach like least-square methods, can be used for the ARX model parameter estimation. Second, we derive the algorithm to compute the open-loop system and Kalman filter Markov parameters from the estimated ARX model parameters. In this step, we first compute the closed-loop system and Kalman filter Markov parameters from the estimated ARX model parameters. Then the open-loop system and Kalman filter Markov parameters are computed from the closed-loop system and Kalman filter Markov parameters and the known controller Markov parameters. Third, the state-space model for the open-loop system is realized from the open-loop Markov parameters through the singular value decomposition method (Chen et al., 1984; Juang and Pappa, 1985). Finally, the Kalman filter for the open-loop system can be estimated from the realized state-space model and the open-loop Kalman filter Markov parameters through a least-square approach.

With this closed-loop identification, an iterative LQG controller design can be performed. Since the Kalman filter used in this LQG controller is obtained directly from the closed-loop identification, it automatically takes into account the effect of the controller on the noise statistics. The LQR tends to reject the process noise and the Kalman filter tends to filter out the measurement noise. Therefore, the closed-loop identification can improve the LQG design and an updated LQG controller can enhance the closed-loop identification in the next cycle. After a certain number of iterations, the LQG controller will converge.

A similar approach is presented by Liu and Skelton (1990). As compared to that approach, this research has the following contributions. First, the proposed method is developed under the *stochastic* framework rather than a deterministic one. Second, the Kalman filter gain is also identified so that it can be used for state estimation directly. Third, random excitation rather than pulse response is used for the closed-loop identification. Finally, since the Kalman filter gain is identified, LQR state feedback is used rather than output feedback. Numerical and experimental results are provided to illustrate and validate this controller design.

2 Closed-Loop State-Space and ARX Models Relationship

Since the relation between the input/output data and the model parameters of a state-space model is nonlinear, parameter estimation of a state-space model from input/output data is a nonlinear programming problem. Nonlinear programming is difficult to solve in general and involves complex iterative numerical methods. The convergence and uniqueness of the solution are also not guaranteed. Unlike a state-space model, the ARX model has a linear relationship between its model parameters and input/output data. Therefore, linear programming can be used for identifying the ARX model. After obtaining the ARX model, a state-space model can be developed based on the relation between these two models. In this section, the relation between a closed-loop state-space model and an ARX model is derived by using z-transforms.

A finite-dimensional, linear, discrete-time, time-invariant system can be modeled as:

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

$$y_k = Cx_k + v_k. \tag{2}$$

where $x \in R^{n\times 1}$, $u \in R^{s\times 1}$, $y \in R^{m\times 1}$ are state, input and output vectors, respectively; w_k is the process noise, v_k the measurement noise; $[A,\ B,\ C]$ are the state-space parameters. Sequences w_k and v_k are assumed gaussian, white, zero-mean, and stationary with covariance matrices W and V, respectively. One can derive a steady-state filter innovation model (Haykin, 1991):

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + AK\varepsilon_k \tag{3}$$

$$y_k = C\hat{x}_k + \varepsilon_k. \tag{4}$$

where $\hat{x}_k$ is the a priori estimated state, K is the steady-state Kalman filter gain and ε_k is the residual after filtering. The existence of K is guaranteed if the system is detectable and $(A, W^{1/2})$ is stabilizable (Goodwin and Sin, 1984). The advantage of using the filter innovation model in the closed-loop identification is that one can directly identify the Kalman filter gain without estimating the covariance matrices of both process and measurement noise which usually are difficult to be obtained from test data (Chen and Huang, 1994).

Similarly, any kind of dynamic output feedback controller can be modeled as:

$$p_{k+1} = A_d p_k + B_d y_k \tag{5}$$

$$u_k = C_d p_k + D_d y_k + r_k, \tag{6}$$

where A_d, B_d, C_d, and D_d are the system matrices of the dynamic output feedback controller, p_k the controller state and r_k is the open-loop input to the closed-loop system. Combining (3) to (6), the augmented closed-loop system dynamics becomes

$$\eta_{k+1} = A_c\eta_k + B_c r_k + A_c K_c \varepsilon_k \tag{7}$$

$$y_k = C_c\eta_k + \varepsilon_k, \tag{8}$$

where

$$A_c = \begin{bmatrix} A + BD_dC & BC_d \\ B_dC & A_d \end{bmatrix},\ B_c = \begin{bmatrix} B \\ 0 \end{bmatrix},\ A_cK_c = \begin{bmatrix} AK + BD_d \\ B_d \end{bmatrix},\ C_c = \begin{bmatrix} C & 0 \end{bmatrix}, \tag{9}$$

and $\eta_k = \begin{bmatrix} \hat{x}_k \\ p_k \end{bmatrix}$. It is noted that K_c can be considered as the Kalman filter gain for the closed-loop system and the existence of the steady-state K_c is guaranteed when the closed-loop system matrix A_c is nonsingular. Substituting (8) into (7) yields

$$\eta_{k+1} = \overline{A}\eta_k + B_c r_k + A_c K_c y_k, \tag{10}$$

where $\overline{A} = A_c - A_c K_c C_c$ and is guaranteed to be asymptotically stable because the steady-state Kalman filter gain K_c exists. The z-transform of (10) and (8) yields

$$\eta(z) = (z - \overline{A})^{-1}(A_c K_c y(z) + B_c r(z)) \tag{11}$$

$$y(z) = C_c \eta(z) + \varepsilon(z). \tag{12}$$

Substituting (11) into (12), one has

$$y(z) = C_c (z - \overline{A})^{-1}(A_c K_c y(z) + B_c r(z)) + \varepsilon(z). \tag{13}$$

The inverse z-transform of (13) with $(z - \overline{A})^{-1} = \sum_{i=1}^{\infty} \overline{A}^{i-1} z^{-i}$ yields

$$y_k = \sum_{i=1}^{\infty} C_c \overline{A}^{i-1} A_c K_c y_{k-i} + \sum_{i=1}^{\infty} C_c \overline{A}^{i-1} B_c r_{k-i} + \varepsilon_k. \tag{14}$$

Since $\overline{A}$ is asymptotically stable, $\overline{A}^i \approx 0$ if $i > q$ for a sufficient large number q. Thus (14) becomes

$$y_k \approx \sum_{i=1}^{q} a_i y_{k-i} + \sum_{i=1}^{q} b_i r_{k-i} + \varepsilon_k \tag{15}$$

where

$$a_i = C_c \overline{A}^{i-1} A_c K_c,\ b_i = C_c \overline{A}^{i-1} B_c. \tag{16}$$

The model described by (15) is the ARX model which directly represents the relationship between the input and output of the closed-loop system. The coefficient matrices a_i and b_i can be estimated through least-square methods from random excitation input r_k and the corresponding output y_k. From (15) by neglecting ε_k, the least-square problem becomes $\xi^T = \begin{bmatrix} a_1 & b_1 & \cdots & a_q & b_q \end{bmatrix} \Phi^T$ or $\xi = \Phi\theta$, where

$$\Phi = \begin{bmatrix} y_q^T & r_q^T & y_{q-1}^T & r_{q-1}^T & \cdots & y_1^T & r_1^T \\ y_{q+1}^T & r_{q+1}^T & y_q^T & r_q^T & \cdots & y_2^T & r_2^T \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ y_{l-1}^T & r_{l-1}^T & y_{l-2}^T & r_{l-2}^T & \cdots & y_{l-q}^T & r_{l-q}^T \end{bmatrix},$$

$\xi = [y_{q+1} \quad y_{q+2} \quad \cdots \quad y_l]^T$, $\theta = [a_1 \quad b_1 \quad a_2 \quad b_2 \quad \cdots \quad a_q \quad b_q]^T$, and l is the number of data points. The integer l has to be large enough so that the Φ matrix has more rows than columns. The batch least-square solution is

$$\theta = (\Phi^T \Phi)^{-1} \Phi^T \xi \tag{17}$$

Therefore, solving for an ARX model simply involves solving a linear programming problem involving an over determined set of equations.

3 Markov Parameters and State-Space Realization

In the previous section, an ARX model, which represents a closed-loop system, is identified from the input/output data through the least-square method. With the known controller dynamics, the estimated ARX model can be transformed to an open-loop state-space model by the following steps. First, the closed-loop system and Kalman filter Markov parameters are calculated from the estimated coefficient matrices of the ARX model. Second, the open-loop system and Kalman filter Markov parameters are derived from the closed-loop system Markov parameters, the closed-loop Kalman filter Markov parameters, and the known controller Markov parameters. Third, the open-loop state-space model is realized by using singular-value decomposition for a Hankel matrix formed by the open-loop system Markov parameters. Finally, an open-loop Kalman filter gain is calculated from the realized state-space model and the open-loop Kalman filter Markov parameters through least-squares.

The z-transform of the open-loop state-space model (3) yields

$$\hat{x}(z) = (z - A)^{-1}(Bu(z) + AK\varepsilon(z)). \tag{18}$$

Substituting (18) to the z-transform of the output equation (4), one has

$$\begin{aligned} y(z) &= C(z - A)^{-1}(Bu(z) + AK\varepsilon(z)) + \varepsilon(z) \\ &= \sum_{k=1}^{\infty} Y(k) z^{-k} u(z) + \sum_{k=0}^{\infty} N(k) z^{-k} \varepsilon(z), \end{aligned} \tag{19}$$

where $Y(k) = CA^{k-1}B$ are the open-loop system Markov parameters; $N(k) = CA^{k-1}AK$, for $k = 1,\ldots,\infty$, open-loop Kalman filter Markov parameters, and $N(0) = I$ which is an identity matrix. Similarly, for the dynamic output feedback controller (5) and (6) and the closed-loop state-space model (7) and (8), one can derive

$$u(z) = \sum_{k=0}^{\infty} Y_d(k) z^{-k} y(z) + r(z) \tag{20}$$

$$y(z) = \sum_{k=1}^{\infty} Y_c(k) z^{-k} r(z) + \sum_{k=0}^{\infty} N_c(k) z^{-k} \varepsilon(z), \tag{21}$$

where $Y_d(0) = D_d$, and $Y_d(k) = C_d A_d^{\,k-1} B_d$, for $k = 1,\ldots,\infty$, are the controller Markov parameters; $Y_c(k) = C_c A_c^{\,k-1} B_c$ the closed-loop system Markov parameters; and $N_c(0) = I$, $N_c(k) = C_c A_c^{\,k-1} A_c K_c$, for $k = 1,\ldots,\infty$, the closed-loop Kalman filter Markov parameters.

Closed-Loop System and Kalman Filter Markov Parameters. The z-transform of the ARX model (15) yields

$$\left(I - \sum_{i=1}^{q} a_i z^{-i} \right) y(z) = \sum_{i=1}^{q} b_i z^{-i} r(z) + \varepsilon(z). \tag{22}$$

Applying long division to (22), one has

$$y(z) = (b_1 z^{-1} + (b_2 + a_1 b_1) z^{-2} + (b_3 + a_1(b_2 + a_1 b_1) + a_2 b_1) z^{-3} + \ldots) r(z) +$$

$$(I + a_1 z^{-1} + (a_1 a_1 + a_2) z^{-2} + (a_1(a_1 a_1 + a_2) + a_2 a_1 + a_3) z^{-3} + \ldots)\varepsilon(z).$$

After comparing with (21), the closed-loop system and Kalman filter Markov parameters can be recursively calculated from the estimated coefficient matrices of the ARX model,

$$Y_c(k) = b_k + \sum_{i=1}^{k} a_i Y_c(k-i) \tag{23}$$

$$N_c(k) = \sum_{i=1}^{k} a_i N_c(k-i). \tag{24}$$

It is noted that $Y_c(0) = 0$, $N_c(0) = I$, and $a_i = b_i = 0$, when $i > q$. One may obtain (23) and (24) from (16) and the definition of the Markov parameters (Phan et al., 1991; Juang et al., 1993). However, the derivation is much more complex.

Open-Loop System and Kalman Filter Markov Parameters. Next, the open-loop system and Kalman filter Markov parameters can be derived from the closed-loop system Markov parameters, the closed-loop Kalman filter Markov parameters, and the known controller Markov parameters. Substituting (20) into (19) yields

$$\begin{aligned} y(z) &= \left(\sum_{k=1}^{\infty} Y(k) z^{-k}\right)\left(\sum_{k=0}^{\infty} Y_d(k) z^{-k} y(z)\right) + \sum_{k=1}^{\infty} Y(k) z^{-k} r(z) + \sum_{k=0}^{\infty} N(k) z^{-k} \varepsilon(z) \\ &= \sum_{k=1}^{\infty} \alpha_k z^{-k} y(z) + \sum_{k=1}^{\infty} Y(k) z^{-k} r(z) + \sum_{k=0}^{\infty} N(k) z^{-k} \varepsilon(z), \end{aligned} \tag{25}$$

where $\alpha_k = \sum_{i=1}^{k} Y(i) Y_d(k-i)$. Rearranging (25), one has

$$\left(I - \sum_{k=1}^{\infty} \alpha_k z^{-k}\right) y(z) = \sum_{k=1}^{\infty} Y(k) z^{-k} r(z) + \sum_{k=0}^{\infty} N(k) z^{-k} \varepsilon(z). \tag{26}$$

Similarly, one can apply long division to (26), and then compare it with (21), to describe the closed-loop system Markov parameters recursively in terms of the open-loop system and the controller Markov parameters,

$$Y_c(j) = Y(j) + \sum_{k=1}^{j} \alpha_k Y_c(j-k) = Y(j) + \sum_{k=1}^{j} \sum_{i=1}^{k} Y(i) Y_d(k-i) Y_c(j-k). \tag{27}$$

And the closed-loop Kalman filter Markov parameters can be recursively expressed in terms of the open-loop system Markov parameters, the open-loop Kalman filter Markov parameters, and the controller Markov parameters as following:

$$N_c(j) = N(j) + \sum_{k=1}^{j} \alpha_k N_c(j-k) = N(j) + \sum_{k=1}^{j} \sum_{i=1}^{k} Y(i) Y_d(k-i) N_c(j-k). \tag{28}$$

Rearranging (27) and (28), one has

$$Y(j) = Y_c(j) - \sum_{k=1}^{j} \sum_{i=1}^{k} Y(i) Y_d(k-i) Y_c(j-k) \tag{29}$$

$$N(j) = N_c(j) - \sum_{k=1}^{j} \sum_{i=1}^{k} Y(i) Y_d(k-i) N_c(j-k). \tag{30}$$

Equations (29) and (30) show that one can recursively calculate the open-loop system and Kalman filter Markov parameters from the closed-loop system Markov parameters in (23), the closed-loop Kalman filter Markov parameters in (24), and the known controller Markov parameters $Y_d(k) = C_d A_d^{k-1} B_d$. It is noted that $Y_c(0) = 0$ and $N_c(0) = I$. One can easily verify (29) and (30) from (9), and also from the definition of the Markov parameters.

State-Space Realization. The open-loop state-space model can be realized from the open-loop system Markov parameters through the Singular Value Decomposition (SVD) method (Chen, 1984; Juang and Pappa, 1985). The first step is to form a Hankel matrix from the open-loop system Markov parameters,

$$H(j) = \begin{bmatrix} Y(j) & Y(j+1) & \cdots & Y(j+\beta) \\ Y(j+1) & Y(j+2) & \cdots & Y(j+\beta+1) \\ \vdots & \vdots & \ddots & \vdots \\ Y(j+\gamma) & Y(j+\gamma+1) & \cdots & Y(j+\gamma+\beta) \end{bmatrix} \tag{31}$$

where $Y(j)$ is the j-th Markov parameter. For a noise free system, if the arbitrary integers

$\beta \geq n$, and $\gamma \geq n$ (the order of the system), the Hankel matrix $H(j)$ is of rank n. From the measurement Hankel matrix, the realization uses the SVD of $H(1)$, $H(1) = U\Sigma V^T$, to identify a n-th order discrete state-space model as

$$A = \Sigma_n^{-1/2} U_n^T H(2) V_n \Sigma_n^{-1/2},\ B = \Sigma_n^{1/2} V_n^T E_s,\ C = E_m^T U_n \Sigma_n^{1/2} \tag{32}$$

where matrix Σ_n is the upper left hand $n \times n$ partition of Σ containing the n largest singular values along the diagonal. Matrices U_n and V_n are obtained from U and V by retaining only the n columns of singular vectors associated with the n singular values. Matrix E_m is a matrix of appropriate dimension having m columns, all zero except that the top $m \times m$ partition is an identity matrix. E_s is defined similarly.

Open-Loop Kalman Filter Gain. Once the open-loop A and C are obtained, one can easily calculate the open-loop Kalman filter gain from the open-loop Kalman filter Markov parameters $N(k) = CA^k K$ in a least-square sense as follows

$$K = (O^T O)^{-1} O^T \begin{bmatrix} N(1) \\ \vdots \\ N(k) \end{bmatrix}, \text{ where } O = \begin{bmatrix} CA \\ \vdots \\ CA^k \end{bmatrix}. \tag{33}$$

The integer k has to be large enough so that the matrix O has more rows than columns. The identified Kalman filter gain can be used directly for state estimation.

4 Iterative LQG Controller Design

Classical LQG controllers are designed by solving two separate, but dual problems: the LQR design and Kalman filter design. Here, the Kalman filter gain can be simultaneously obtained with the open-loop state-space model through the closed-loop identification. Only the LQR design based on the identified open-loop model needs to be solved. The performance index for the LQR problem is defined as

$$P.I. = \sum_{k=1}^{\infty} y_k^T Q y_k + u_k^T R u_k = \sum_{k=1}^{\infty} x_k^T C^T Q C x_k + u_k^T R u_k \tag{34}$$

where weighting matrices Q and R are design parameters. We can summarize the iterative LQG controller design as follows:

1. Use the a priori open-loop model and arbitrary covariance matrices of the measurement and process noise to design the LQR and Kalman filter. Then, calculate the controller Markov parameters. The weighting matrices Q and R for the LQR chosen here will remain the same in the following iterations.
2. Apply random excitation input to the closed-loop system and record the closed-loop input/output data.
3. Estimate the coefficient matrices of the closed-loop ARX model by using (17).
4. Calculate the closed-loop system and Kalman filter Markov parameters by using (23) and (24).
5. Calculate the open-loop system and Kalman filter Markov parameters by using (29) and (30).
6. Realize the open-loop state-space system matrices $\left[\hat{A}, \quad \hat{B}, \quad \hat{C}\right]$ by using (31) and (32).
7. Estimate the open-loop Kalman filter gain $\hat{K}$ by using (33).
8. Obtain the LQR feedback gain F by solving the corresponding Riccati equation based on the identified open-loop model.
9. Form the updated LQG controller in (5) and (6) by using $A_d = \hat{A} - \hat{B}F - \hat{A}\hat{K}\hat{C}$, $B_d = \hat{A}\hat{K}$, $C_d = -F$, and $D_d = 0$.
10. Calculate the updated controller Markov parameters and check the convergence of the controller by

$$\delta = \sum_{k=0}^{n} \left\| Y_d(k)_{updated} - Y_d(k)_{previous} \right\|_2 . \qquad (35)$$

If δ is greater than a desired value, go back to step 2, otherwise stop.

5 Numerical and Experimental Example

The proposed iterative LQG controller design has been applied to control design of the Large-Angle Magnetic Suspension Test Facility (LAMSTF) (Groom and Britcher, 1992; Groom and Schaffner, 1990) developed in NASA Langley Research Center (see Fig. 1). The LAMSTF is a laboratory-scale research project to demonstrate the magnetic suspension of objects over wide ranges of attitudes. This system represents a scaled model of a planned Large-Gap Magnetic Suspension System. The LAMSTF system consists of a planar array of five copper electromagnets which actively suspend a small cylinder with a permanent magnet core. The cylinder is a rigid body and has six independent degrees of freedom, namely, three displacements (x, y and z) and rotations (pitch, yaw and roll). Currents in the electromagnets generate a magnetic field which produces a net force and torque on the suspended cylinder. The roll of the cylinder is uncontrollable, and is assumed to be motionless. Five pairs of the LEDs and light receivers are used to indirectly sense the pitch and yaw angles, and three displacements of the cylinder's centroid. Therefore, the control inputs to the system consist of five currents sent into five electromagnets and the system outputs are five voltage signals measured from five photo detectors. The forces on the cylinder are, in general, non-linear functions of space and current. Therefore, only the linear time-invariant perturbed motion about an equilibrium state is considered. Because it is difficult to accurately model the magnetic field and its gradients, the analytical model contains some modeling errors. Therefore, the performance of the LQG controller based on the analytical model alone is unsatisfactory.

The system matrices of the analytical model are shown in the appendix. The eigenvalues of the system matrix indicate that the LAMSTF system includes highly unstable real poles (about 10 Hz) and low-frequency oscillatory modes (about 0.16 Hz). For both numerical simulation and experiment, the sampling rate is 250 Hz. The performance index used for the LQR design is also shown in the appendix. The step command for all simulations and experiments is 0.02 radian for pitch and yaw, and 0.2 mm for x, y, and z.

In the numerical simulation, the analytical model is used as the true model. In each iteration, the ratios of the process and measurement noise to the corresponding signal are 2% and 1%, respectively. To simulate modeling error and unknown noise statistics, the initial LQG controller is designed by using an altered model of which each parameter is 5% greater than the corresponding parameter of the analytical model and guessed covariance matrices of noise $W = 10I_{10\times10}$ and $V = I_{5\times5}$. The simulated step response with this initial controller for the pitch, yaw, x, y, and z is shown in Fig. 2. It is clear that the result is very poor. After performing the first iteration of the proposed iterative LQG controller design, the step response shown in Fig. 3 is greatly improved. The performance is further improved slightly in the following iterations. Figure 4 shows how the controller converges by comparing the (1,1) element of the controller Markov parameters.

For a noise free system, the exact open-loop model can be obtained after the first closed-loop identification and no further iteration is required. In this case, the identified Kalman filter gain becomes the dead-beat observer gain (Phan et al., 1991; Juang et al., 1993). For a noise corrupted system, iterations are required to update the open-loop model and the Kalman filter gain until the iterative LQG controller converges. Although the numerical simulations show that the iterative controller can converge quickly, the required conditions to guarantee the convergence need further study.

In the experiments, the analytical model and guessed covariance matrices of noise $W = 10I_{10\times10}$ and $V = I_{5\times5}$ are used to design the initial LQG controller. The experimental step response with this initial controller is also very poor. The experimental step responses for the first three iterations are compared in Fig. 5 to demonstrate how the step response is improved with iteration. In each iteration, the open-loop system model and the Kalman filter gain are updated through the closed-loop identification from experimental data. The experimental step response improves with each iteration, similar to the simulated cases. The experimental steady-state errors, however, do not go to zero in each case. This is due to drift in the sensor zero between experiments. The system's dynamics have been found to be insensitive to these small changes in the operating point. The results show that the proposed iterative LQG controller design is very effective for controlling this highly unstable magnetic suspension system.

6 Conclusion

In contrast to most existing LQG controller designs of which the great majority solve two separate, but dual problems: the LQR and Kalman filter design, this report proposes an iterative LQG controller design approach. A closed-loop identification method is developed to update the open-loop state-space model and the Kalman filter gain simultaneously from the closed-loop input/output test data. The method is derived under the stochastic framework, taking into account the effects of process noise as well as measurement noise. For a noise free system, the exact open-loop model can be obtained after the first closed-loop identification and the identified Kalman filter gain becomes the dead-beat observer gain. For a noise corrupted system, iterations are required to update the open-loop model and the Kalman filter gain from testing until the iterative LQG controller converges. In each iteration, since the Kalman filter gain is identified directly from test data, the LQG design is simplified to be an LQR design. A highly unstable large-angle magnetic suspension system is used to validate this controller design. Both numerical simulations and test data show that the controller converges quickly and is very effective when the system is subjected to modeling error and unknown noise statistics.

References

Chen, C. T., 1984, *Linear System Theory and Design*, Second Edition, Chapter 6, CBS College Publishing, New York.

Chen, C.-W., Huang, J.-K., Phan, M., and Juang, J.-N., 1992a, "Integrated System Identification and Modal State Estimation for Control of Flexible Space Structures," *AIAA J. Guidance, Control and Dynamics,* Vol. 15, No.1, pp. 88-95.

Chen, C.-W., Huang, J.-K., and Juang, J.-N., 1992b, "Identification of Linear Stochastic Systems Through Projection Filters", in *Proc. AIAA Structures, Structural Dynamics and Materials Conf.*, pp. 2330-2340, also to appear in *AIAA J. Guidance, Control and Dynamics.*

Chen, C.-W., Juang, J.-N., and Huang, J.-K., 1993, "Adaptive Linear System Identification and State Estimation," in *Control and Dynamic Systems: Advances in Theory and Applications, Vol. 57, Multidisciplinary Engineering Systems: Design and Optimization Techniques and Their Application*, edited by C. T. Leondes, Academic Press, Inc., pp. 331-368.

Chen, C.-W., Huang, J.-K., 1994, "Estimation of Steady-State Optimal Filter Gain from Non-Optimal Kalman Filter Residuals," *ASME J. Dynamic Syst., Meas., and Contr,* Vol. 116, No. 3, pp. 550-553.

Goodwin, G. C., and Sin, K. S., 1984, *Adaptive Filtering, Prediction and Control*, Prentice-Hall, Englewood Cliffs, New Jersey 07632.

Groom, N. J., and Britcher, C. P., 1992, "Open-Loop Characteristics of Magnetic Suspension Systems Using Electromagnets Mounted in a Planar Array," NASA-TP 3229.

Groom, N. J., and Schaffner, R. P., 1990, "An LQR Controller Design Approach for a Large Gap Magnetic Suspension System (LGMSS)," NASA TM-101606.

Haykin, S., 1991, *Adaptive Filter Theory*, Second edition, Prentice-Hall, Englewood Cliffs, New Jersey 07632.

Juang, J.-N., and Pappa, R. S., 1985, "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction," *AIAA J. Guidance, Control, and Dynamics,* Vol. 8, pp. 620-627.

Juang, J.-N., Phan, M., Horta, L. G., and Longman, R. W., 1993, "Identification of Observer/Kalman Filter Markov Parameters: Theory and Experiments," *AIAA J. Guidance, Control and Dynamics*, Vol. 16, No. 2, pp. 320-329.

Liu, K., and Skelton, R. E., 1990, "Closed-Loop Identification and Iterative Controller Design," in *Proc. IEEE Int. Conf. Decision and Control*, pp. 482-487.

Phan, M., Horta, L. G., Juang, J.-N., and Longman, R. W., 1993, "Linear System Identification via an Asymptotically Stable Observer", *J. Optimization Theory and Application,* Vol. 79, No. 1, 1993, pp. 59-86.

Phan, M., Juang, J.-N., Horta, L. G., and Longman, R. W., 1992, "System Identification from Closed-Loop Data with Known Output Feedback Dynamics," *J. Guidance, Control and Dynamics*, Vol. 17, No. 4, 1994, pp. 661-669.

Appendix

The analytical model of the large-angle magnetic suspension test facility is

$$\dot{x} = A_m x + B_m u \tag{A1}$$

$$y = C_m x \tag{A2}$$

where $x = \begin{Bmatrix} x_p \\ \dot{x}_p \end{Bmatrix}$, $A_m = \begin{bmatrix} 0_{5\times5} & I_{5\times5} \\ A_{21} & A_{22} \end{bmatrix}$, $B_m = \begin{bmatrix} 0_{5\times5} \\ B_2 \end{bmatrix}$ and $C_m = \begin{bmatrix} C_1 & 0_{5\times5} \end{bmatrix}$. The state variable x_p includes pitch and yaw angles and three linear displacements of the cylinder's centroid. The matrices A_{21}, A_{22}, B_2 and C_1 are

$$A_{21} = \begin{bmatrix} 3.3415e+03 & 0 & -3.9392e+04 & 4.9534e-12 & 2.0811e-12 \\ 0 & 3.3415e+03 & -4.9534e-12 & 4.8609e-12 & -1.4472e-11 \\ -9.8070e+00 & -2.4664e-15 & 4.9937e+01 & 4.3604e-15 & -2.5089e-02 \\ -3.6031e-15 & 1.9618e-15 & 4.3604e-15 & 9.5577e+01 & -9.0007e-15 \\ -2.3357e-16 & -3.6031e-15 & -2.5089e-02 & -9.0007e-15 & -9.1324e-01 \end{bmatrix},$$

$A_{22} = 0_{5\times5}$,

$$B_2 = \begin{bmatrix} 3.8370e+01 & 3.8370e+01 & 3.8370e+01 & 3.8370e+01 & 3.8370e+01 \\ 0 & 8.9802e+01 & 5.5514e+01 & -5.5514e+01 & -8.9802e+01 \\ 2.2144e-01 & -1.5274e-01 & 7.8453e-02 & 7.8453e-02 & -1.5274e-01 \\ 0 & 1.2154e-01 & -1.9674e-01 & 1.9674e-01 & -1.2154e-01 \\ -2.7672e-01 & -8.5465e-02 & 2.2388e-01 & 2.2388e-01 & -8.5465e-02 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 8.9024e+01 & 0 & 0 & 0 & 6.0976e+03 \\ 0 & 0 & 7.8740e+03 & 0 & 0 \\ -1.1625e+02 & 0 & 0 & 0 & 6.2500e+03 \\ 0 & 9.5425e+01 & 0 & -6.5359e+03 & 0 \\ 0 & -1.0725e+02 & 0 & -5.1813e+03 & 0 \end{bmatrix}.$$

The eigenvalues of the system matrix A_m are ± 58.78, ± 57.81, ± 9.78, $\pm j7.97$, and $\pm j0.96$. The matrix C_1 which relates the sensor output voltage to the displacement can be obtained from calibration and is assumed known. To recover the displacement from the sensor output voltage, one can use $x_p = C_1^{-1} y$.

The performance index for the state feedback design is chosen as

$$P.I. = \sum_{k=1}^{\infty} y_k^T Q y_k + u_k^T R u_k \tag{A3}$$

where $Q = (C_1^{-1})^T diag\begin{bmatrix} 1.e3 & 1.e3 & 2.e8 & 2.e8 & 2.e8 \end{bmatrix} C_1^{-1}$ and $R = I_{5\times5}$.

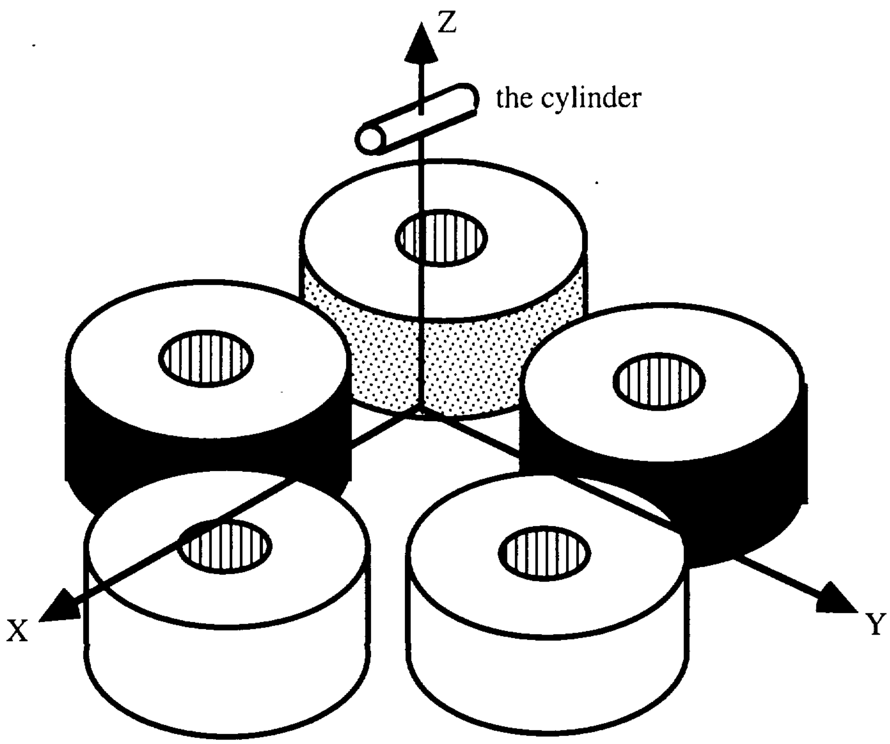

Fig. 1 Large-Angle Magnetic Suspension Test Facility (LAMSTF) Configuration

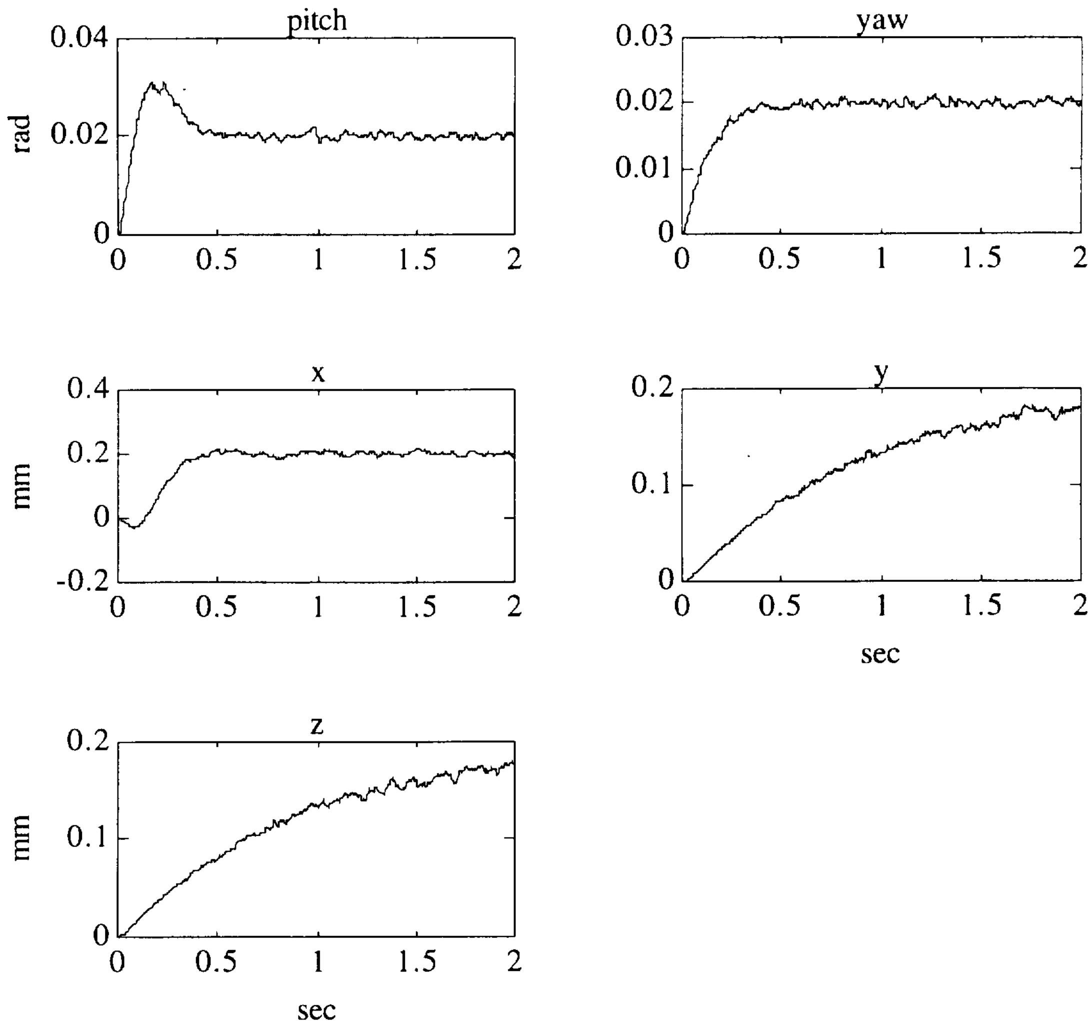

Fig. 2 Simulated step response with the initial LQG controller.

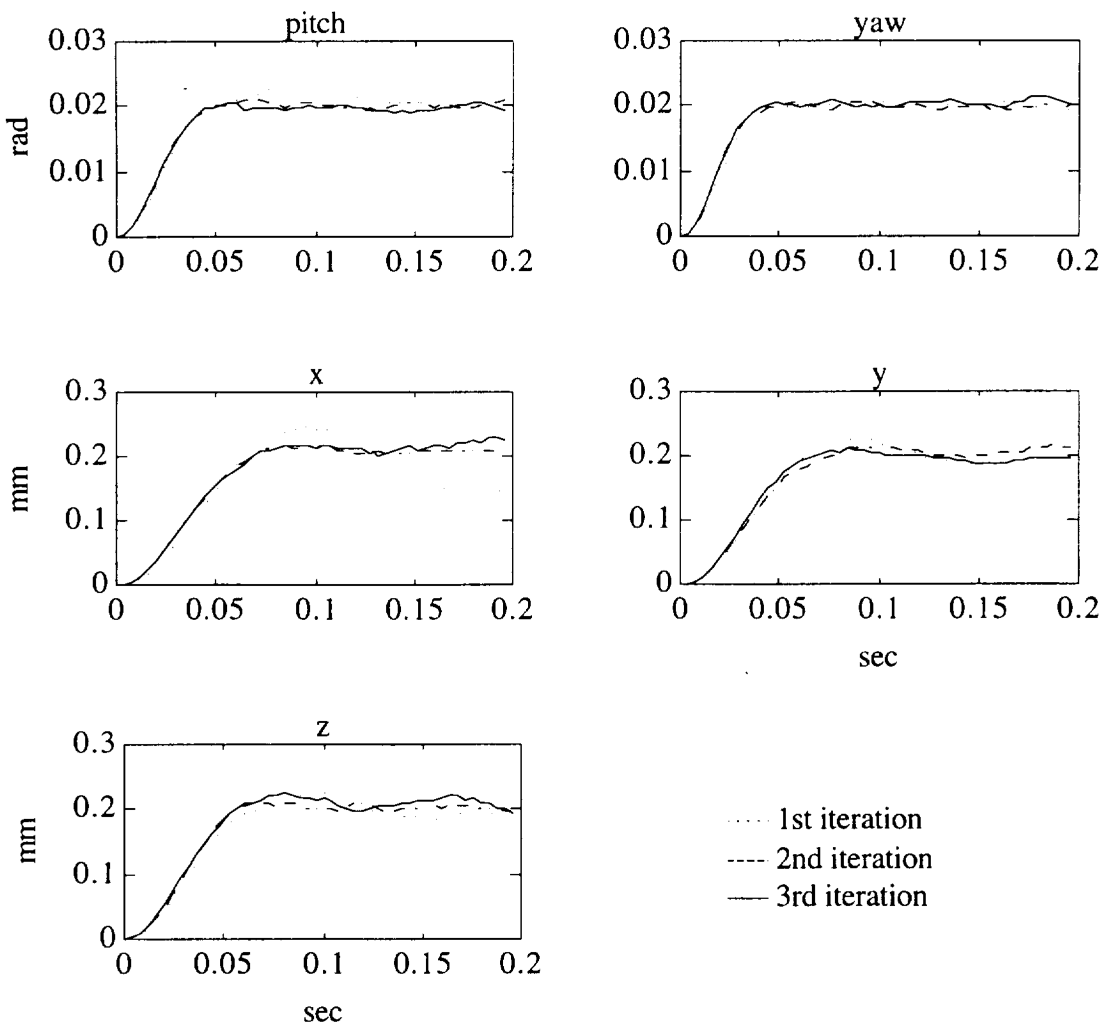

Fig. 3 Comparison of the simulated step response with the iterative LQG controller.

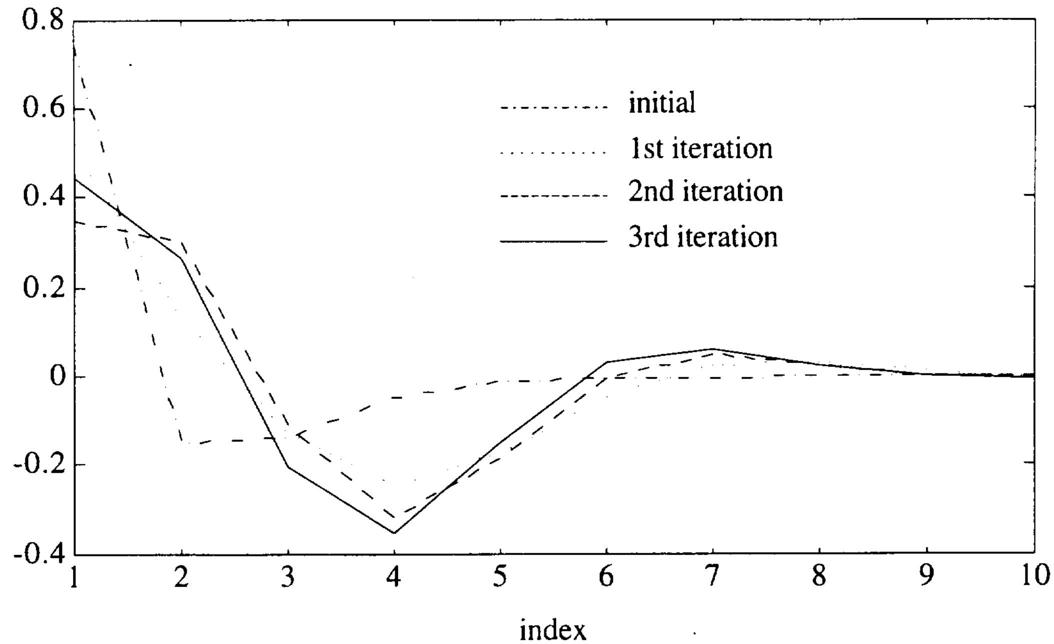

Fig. 4 Comparison of the (1,1) element of the controller Markov parameters

$$Y_d(k) = C_d A_d^{k-1} B_d.$$

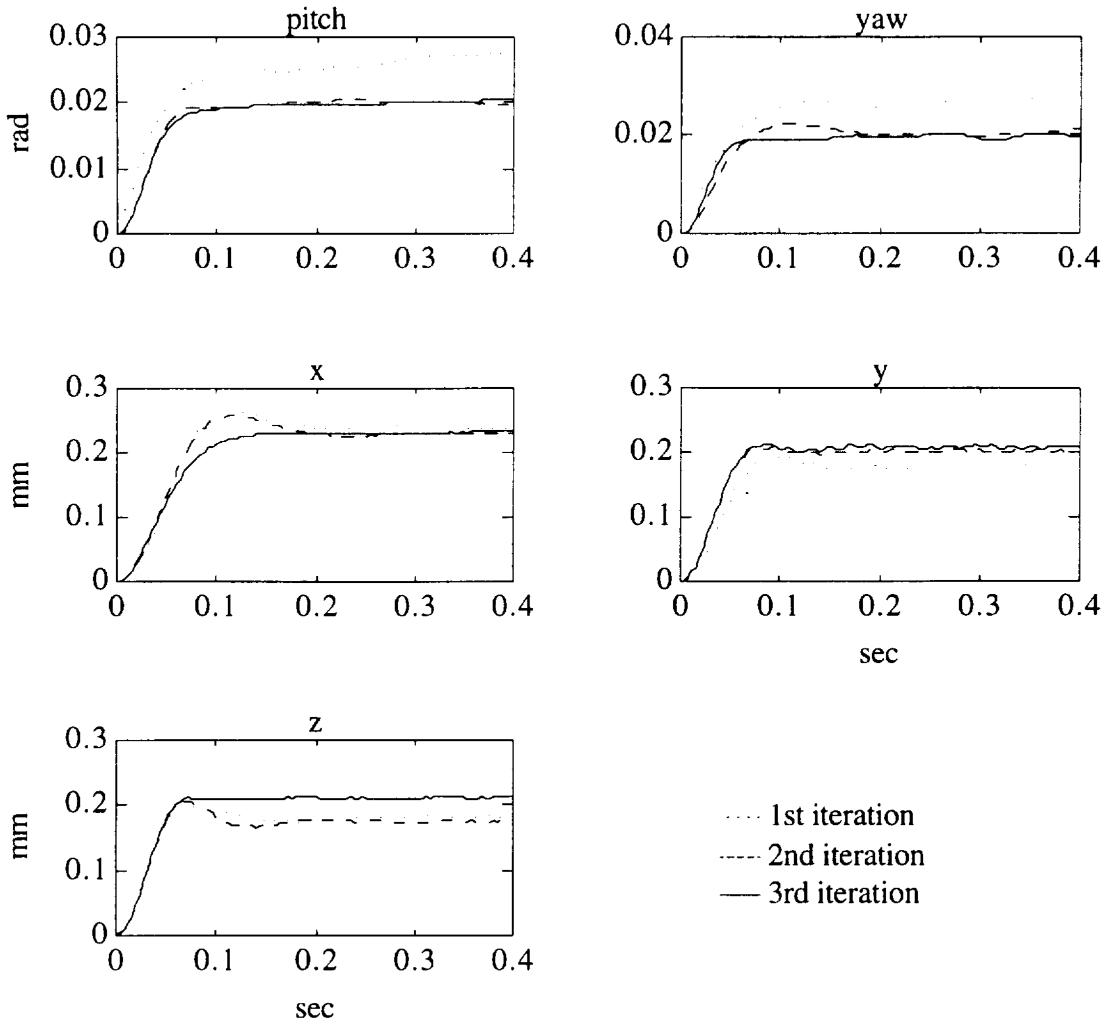

Fig. 5 Comparison of the testing step response with the iterative LQG controller.

PRESENTATIONS

Current Year

Neff, D.J.; Britcher, C.P.: Design and implementation of a digital controller for a vibration isolation and vernier pointing system. 3rd International Symposium on Magnetic Suspension Technology, Tallahassee, Florida, December 1995. NASA CP-3339, July 1996.

Cox, D.E.; Groom, N.J.; Hsiao, M.H.; Huang, J.K.: Modeling and identification of a large gap magnetic suspension system. 3rd International Symposium on Magnetic Suspension Technology, Tallahassee, Florida, December 1995. NASA CP-3339, July 1996.

Britcher, C.P.; Groom, N.J.: Computational analysis of static and dynamic behaviour of magnetic suspensions and magnetic bearings. 3rd International Symposium on Magnetic Suspension Technology, Tallahassee, Florida, December 1995. NASA CP-3339, July 1996.

Britcher, C.P.: Application of magnetic suspension and balance systems to ultra-high Reynolds number facilities. International Workshop on High Reynolds Number Flows. Brookhaven National Laboratory, June 1996 (presented by Dr. Robert A. Kilgore)

Britcher, C.P.: Application of magnetic suspension technology to large scale facilities - progress, problems and promises. To be presented in the AIAA 35th Aerospace Sciences Meeting, Reno, NV, January 1997.

PUBLICATIONS

Current Year

Hsiao, M.H.; Huang, J.K.; Cox, D.E.: Iterative LQG controller design through closed-loop identification. ASME Journal of Dynamic Systems, Measurement and Control, June 1996

Groom, N.J.; Britcher, C.P. (editors): Third International Symposium on Magnetic Suspension Technology. NASA CP-3336, July 1996.

Huang, J.K.; Hsiao, M.H.; Cox, D.E.: Indirect identification of linear stochastic systems with known feedback dynmaics. AIAA Journal of Guidance, Control and Dynmaics, July/August 1996

Lee, H.C.; Hsiao, M.H.; Huang, J.K.; Chen, C.W.: Identification of stochastic system and controller via projection filters. ASME Journal of Vibration and Acoustics, August 1996

Huang, J.K.; Lee, H.C.; Schöen, M.P.; Hsiao, M.H.: State-space system identification from closed-loop frequency response data. Accepted for AIAA Journal of Guidance, Control and Dynamics

SUMMARY INFORMATION

Grant Duration : October 1990 thru November 1996

Funding Level : $416,813

Major Accomplishments :
Development of the LAMSTF experiment. Demonstration of the first LAMSTF controller and development of a general-purpose digital controller. Support of the 6-dof derivative version. Design guidance for ACTF. Recommissioning of the Annular Suspension and Pointing Systems with a digital controller. Systematic analysis and modelling procedures for eddy currents. Development and demonstration of real-time system identification procedues. Support of 3 International Symposia and 1 Workshop. 3 "on-site" graduate students completed with 1 ongoing. Additonal graduate students supported off-site.

Graduate Students working on-site at LaRC :
Mehran Ghofrani, MS in Mechanical Engineering. Currently employed by SatCon technology Corporation. Lucas Foster, MS in Mechanical Engineering. Currently employed by J.J. Mullins and Associates. Daniel J. Neff, MS in Aerospace Engineering. Currently employed by Oxford Superconductors. Dale Bloodgood, MS in Engineering Mechanics (pending).

Additional support was provided for several students working at ODU, notably Marco Schoen and Chien-Hsun Kuo

PRESENTATIONS

Cumulative

Kilgore, W.A.; Britcher, C.P.; Haj, A.: A comparison of digital controllers used with magnetic suspension systems. 1991 ROMAG Conference, Washington D.C., March 1991.

Groom, N.J.; Britcher, C.P.: Stability considerations for magnetic suspension with electromagnets mounted in a planar array. NASA Workshop on Aerospace Applications of Magnetic Suspension Technology, LaRC, Sept 1990. NASA CP-10066, March 1991.

Britcher, C.P.; Ghofrani, M.; Britton, T.C.; Groom, N.J.: The large-angle magnetic suspension test fixture. International Symposium on Magnetic Suspension Technology. NASA LaRC, August 1991. NASA CP-3152, May 1992.

Britcher, C.P.: Large-gap magnetic suspension systems. International Symposium on Magnetic Suspension Technology. NASA LaRC, August 1991. NASA CP-3152, May 1992.

Britcher, C.P.; Ghofrani, M.; Haj, A.; Britton, T.C.: Analysis, modelling and simulation of the large-angle magnetic suspension test fixture. 3rd International Symposium on Magnetic Bearings, Washington D.C., July 1992.

Groom, N.J.; Britcher, C.P.: A Description of a laboratory scale magnetic suspension system with large angular capability. 1st IEEE Conference on Control Applications, Dayton, OH, September 1992.

Britcher, C.P.: Some further developments in the dynamic modelling and control of the large angle magnetic suspension test fixture. 2nd International Symposium on Magnetic Suspension Technology, Seattle, WA, August 1993

Britcher, C.P.: Dynamic modelling of a laboratory model magnetic suspension test fixture with large angular capability. PICAST'1, December 1993

Britcher, C.P.; Groom, N.J.: Current and future development of the annular suspension and pointing system. 4th International Symposium on Magnetic Bearings, August 1994

Britcher, C.P.; Groom, N.J.: Eddy current computations applied to magnetic suspensions and magnetic bearings. MAG-95, August 1995.

Neff, D.J.; Britcher, C.P.: Design and implementation of a digital controller for a vibration isolation and vernier pointing system. 3rd International Symposium on Magnetic Suspension Technology, Tallahassee, Florida, December 1995. NASA CP-3339, July 1996.

Cox, D.E.; Groom, N.J.; Hsiao, M.H.; Huang, J.K.: Modeling and identification of a large gap magnetic suspension system. 3rd International Symposium on Magnetic Suspension Technology, Tallahassee, Florida, December 1995. NASA CP-3339, July 1996.

Britcher, C.P.; Groom, N.J.: Computational analysis of static and dynamic behaviour of magnetic suspensions and magnetic bearings. 3rd International Symposium on Magnetic Suspension Technology, Tallahassee, Florida, December 1995. NASA CP-3339, July 1996.

Britcher, C.P.: Application of magnetic suspension and balance systems to ultra-high Reynolds number facilities. International Workshop on High Reynolds Number Flows. Brookhaven National Laboratory, June 1996 (presented by Dr. Robert A. Kilgore)

Britcher, C.P.: Application of magnetic suspension technology to large scale facilities - progress, problems and promises. To be presented in the AIAA 35th Aerospace Sciences Meeting, Reno, NV, January 1997.

PUBLICATIONS

Cumulative

Groom, N.J.; Britcher, C.P.: Open-loop characteristics of magnetic suspension systems with electromagnets mounted in a planar array. NASA TP-3229, September 1992.

Britcher, C.P.; Ghofrani, M.: A magnetic suspension system with a large angular range. Review of Scientific Instruments, July 1993.

Hsiao, M.H.; Huang, J.K.; Cox, D.E.: Iterative LQG controller design through closed-loop identification. ASME Journal of Dynamic Systems, Measurement and Control, June 1996

Groom, N.J.; Britcher, C.P. (editors): Third International Symposium on Magnetic Suspension Technology. NASA CP-3336, July 1996.

Huang, J.K.; Hsiao, M.H.; Cox, D.E.: Indirect identification of linear stochastic systems with known feedback dynmaics. AIAA Journal of Guidance, Control and Dynmaics, July/August 1996

Lee, H.C.; Hsiao, M.H.; Huang, J.K.; Chen, C.W.: Identification of stochastic system and controller via projection filters. ASME Journal of Vibration and Acoustics, August 1996

Huang, J.K.; Lee, H.C.; Schöen, M.P.; Hsiao, M.H.: State-space system identification from closed-loop frequency response data. Accepted for AIAA Journal of Guidance, Control and Dynamics

APPENDIX

Copies of selected recent publications are included here for reference :

1. AIAA 97-0346. Britcher, C.P.: Application of magnetic suspension technology to large scale facilities - progress, problems and promises. To be presented in the AIAA 35th Aerospace Sciences Meeting, Reno, NV, January 1997.

2. Britcher, C.P.: Application of magnetic suspension and balance systems to ultra-high Reynolds number facilities. International Workshop on High Reynolds Number Flows. Brookhaven National Laboratory, June 1996. Submitted for consideration in the conference Proceedings.

3. Huang, J.K.; Hsiao, M.H.; Cox, D.E.: Indirect identification of linear stochastic systems with known feedback dynmaics. AIAA Journal of Guidance, Control and Dynmaics, July/August 1996

4. Hsiao, M.H.; Huang, J.K.; Cox, D.E.: Iterative LQG controller design through closed-loop identification. ASME Journal of Dynamic Systems, Measurement and Control, June 1996

5. Huang, J.K.; Lee, H.C.; Schöen, M.P.; Hsiao, M.H.: State-space system identification from closed-loop frequency response data. Accepted for AIAA Journal of Guidance, Control and Dynamics